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Case Report

A Case Study on the Development of Math Competence in an Eight-year-old Child with Dyscalculia: Shared Intentionality in Human-Computer Interaction for Online Treatment Via Subitizing

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Abstract

Studies in the field of neuroscience have shown that the neural network responsible for numeracy overlaps with the visual and spatial processing regions. Other studies in psychology also highlighted an association of visual-spatial processing with mathematical competence at the early stages of development. These findings suggest that research on the size of the focal area of attention (consciousness) can contribute to understanding the development of numeracy. In this case study, we verified the hypothesis of developing numeracy in children by training the rapid apperception of a few items called “subitizing.” Shared intentionality promotes cognition from the onset. Therefore, in this study, we investigated this interaction modality to give an eight-year-old girl an insight into expanded apperception of an array in "subitizing" for improving her numerical competence. The child was stimulated to apperceive more objects while performing “subitizing” tasks with the mother. The course of treatment consisted of the four regimes of human-computer interaction based on rapid exposure to several pictures with a few dots. Simultaneously, this human-computer interaction also stimulated shared intentionality in the mother-child dyad for developing the child’s rapid apprehension of these small quantities. The outcome of this intervention was an increase in the size of the focal point of attention (consciousness) and the development of numerical competence, where an association was established between the expanding apperception and the developing numeracy.

Keywords
Dyscalculia; e-learning; human-computer interaction; mathematical learning disability; mathematical difficulties; shared intentionality; subitizing

1. Introduction

We conducted this interdisciplinary study to understand the development of numeracy in children using a new approach to address the issue. The study was novel in evaluating human-computer interaction for stimulating shared intentionality in the mother-child dyad to expand the size of the focal area of attention in the child. Introducing this new approach requires an understanding of Dyscalculia and an analysis of studies that consider the neurobiological basis of shared intentionality and how the latter can be stimulated by human-computer interactions. In the Introduction section, we discussed the theoretical and empirical grounds based on which this study was conducted. Specifically, to elucidate the knowledge gap, in the Introduction, we briefly discussed the theoretical background on the causes and interventions of Dyscalculia. Then, we showed, for the first time, the empirical data analysis on developmental stages of the focal area of attention in children based on the data on subitizing collected over a hundred years. Several studies found an association between the apprehension of dots and numerical competence, as well as that dyscalculic children, showed deficits in subitizing. The results also showed that in many cases, Dyscalculia develops in children with a typical developmental trajectory when they cannot provide attention to more than three items. The advanced representation and mental manipulation capabilities of an array can increase numerical competence. Increasing the size of the focal area of attention facilitates the development of numeracy in children. Next, we presented empirical data that addressed a hypothesis regarding the neurobiological basis of shared intentionality that occurs in non-perceptual interaction. Finally, we discussed the hypothesis on how human-computer interactions can stimulate shared intentionality for the child's apperception of more dots in subitizing than the child can apprehend. Thus, shared intentionality can provide an insight into arithmetic principles and memorizing relationships between numbers needed for math competence.

Dyscalculia or mathematical learning disability (MLD) is a neuro-developmental disorder that refers to difficulties in understanding math problems (e.g., [1, 2]). Mathematical learning difficulties (MD) are mathematical deficits comprising dyscalculia (or MLD) and math deficits that may be caused by non-neurobiological factors (e.g., [1-4]). About 5-9% of school-age children are affected by MD [5-7]. Children with difficulties in mathematics are in many respects similar to their peers without MD; children with MD were found to have "essential cognitive normality" [5]. Children with MLD do not have problems with brain activity; instead, their brain works differently [8]. These disabilities may develop in children with normal IQ in the absence of difficulties in other domains, skills, or abilities [1, 9]. Children who have difficulties in mathematics usually have difficulty remembering number facts [5, 7, 10-12]. A study [13] found that 3-7% of children, adolescents, and adults have MLD or dyscalculia. MD leads to developmental delay since many schools assume that eight-year-old children possess knowledge of basic arithmetic facts.

Dehaene [14] proposed that dyscalculia might be associated with a deficit in ‘‘number sense”, which underlies our ability to quickly understand, approximate, and manipulate numerical quantities non-verbally. A slightly different hypothesis was proposed by Butterworth [15], who postulated that children with developmental dyscalculia might suffer from a ‘‘defective number module” induced by a highly selective impairment of the capacity to understand and represent numerosity which in turn leads to difficulties in learning arithmetic.

According to Amalric [16], a math-responsive network is established in the bilateral intraparietal sulcus and the inferior temporal cortex. This network is distinct from language processing networks but overlaps with visual and spatial processing regions [16]. This network overlaps with the neural correlates of deductive reasoning [16]. A meta-analysis suggested that the left parietal cortex is more consistently activated while processing symbolic quantities (visually-presented Arabic digits), whereas activity in the right parietal cortex is more reliably associated with the processing of nonsymbolic formats [17]. However, the exact causes for developmental dyscalculia are unknown, though studies have shown that they might be due to issues in brain development and genetics (as the disability tends to run in families) [18].

The number of intervention studies has increased dramatically in recent years [19]. A comprehensive overview included 41 review articles and meta-analyses, i.e., a metareview, targeting early numeracy or more advanced mathematics, focusing on both typical and atypical (or at risk) achievers in preschool, primary, or secondary education [19]. However, only low-efficiency cognitive-based interventions are available for treating children and adolescents with MLD [20]. Children do not have a network of rules and principles to associate number combinations; they have to memorize many isolated facts [10, 21-23]. It requires remembering [23-25] at least 45 basic addition facts (3-18) with addends up to 9 and answers up to 18. In modern teaching, thinking strategies have been implemented to help children (a) learn numerical relationships (e.g., [10, 26]) and (b) foster the automatic recall of basic number facts (e.g., [10, 24-26]). However, memorizing basic arithmetic facts is an unpredictable process due to the lack of efficient strategies to memorize specific numerical associations. Modern teaching models represent basic arithmetic facts as a network of many specific numerical bonds. Mastery of arithmetic facts includes explicit drill memorizing (by auditory and visual inputs) and implicit insight into associativity and commutativity principles, and memorizing relationships between numbers.

Although arithmetic difficulties are widespread, the predictable strategies for memorizing basic arithmetic facts are understudied. Current theories suggest that the efficient production of number combinations is exclusively a reproductive process when basic combinations are committed to long-term memory [10].

Wundt suggested that “the size of the focal area of consciousness” is three or four items (Wundt presented arrays of Latin letters) for “unpracticed observers,” but it can be expanded to six with practice [27]. Leahey [27] argued that Wundt’s theory of attention is a viable alternative to contemporary formulations; it received as much empirical support as any modern theory [27]. The Wundtian outcome was one of the earliest findings that provided a method for studying the “range of attention.”

Other researchers adopted the Wundtian visual mode research of the focal point of attention (consciousness) and introduced several modifications compatible with the sense of this approach. Freeman [28] investigated "the scope of attention," applying the Wundtian research paradigm while changing the stimulus. To determine the scope of attention in its simplest form, Freeman [28] presented several similar objects (several dots) to a small group of subjects. He registered the difference between the number of objects (dots) that may be grasped simultaneously in a single observation event by adults and children of various ages. The results showed that children (below eight years) could judge the number correctly up to four items.

In this study, we presented the origins of the mode of research on the accuracy of estimating the number of displayed dots under time pressure to highlight that it was initially developed to observe visual apprehension- or the "range of attention" (e.g., [29]). Later, this approach was used for understanding the development of numerical competence (e.g., [30-37]). The experiments on numeracy demonstrated that response accuracy was high when the number of dots did not exceed six. Kaufman et al. [30] proposed the term "subitizing" for this rapid apprehension of small quantities. The experiments on subitizing studied numeracy. However, they also determined the size of the focal point of consciousness, although the researchers did not intend to do so. For studying numerical competence in subjects, these studies applied the main ideas of the Wundtian research paradigm, which requires examination of the visual size of the focal area of consciousness. From this perspective, the purpose of the experimental paradigm on subitizing is similar to that proposed by Wundt and Freeman, i.e., this research mode (on the accuracy of estimating the number of dots in a display under time pressure) determined the visual focal point of consciousness while collecting and analyzing the data to interpret other parameters.

Analysis of the studies on subitizing might help to determine the developmental stages of the focal area of consciousness in children. The empirical data on subitizing has four stages:

i) The first stage with up to two items in the focal point lasts up to about six months of age (e.g., [32, 37]). This conclusion is based on the fact that six-month-old infants can discriminate numerosities with a 1:2 ratio but not a 2:3 ratio [37].

ii) The second stage with up to three items probably begins after eight months (e.g., [32, 34-36]). Ten-and 12-month-old infants can track up to three items precisely, but not four or more objects (e.g., [35, 36]). This suggests that three objects might be captured by children's focal point of consciousness at this stage. Furthermore, experiments have shown that the focal point size in even two-year-old children is up to 3 (e.g., [32, 34]).

iii) The third stage appears at about 3.5 years of age with four items. Some studies (e.g., [34, 38]) have suggested that the size of the focal point of consciousness might be up to four items at this stage.

iv) The fourth stage starts in children when they are about five years old and can hold five or more items in the focal point (e.g., [34]).

Differences in cognition rely on different abilities in cognitive functions such as memory, learning, the use of language, problem-solving, decision making, reasoning, and intelligence [39]. General cognitive functions in preschoolers are associated with math abilities [7, 38, 40-42]. Cognitive functions such as learning, problem-solving, decision making, reasoning, and intelligence rely, at least partly, on the comparison, association, and categorization of different phenomena simultaneously kept in the focal point of attention. The involvement of more objects in mental operation is associated with greater development of mental processing skills and more possibilities for apprehension, judgment, and reasoning. For example, three items in the focal point of consciousness have six possible combinations (3 factorial), and four items have 24 (4 factorial) combinations. This difference becomes significant in the case of a focal point with six items with 720 possible combinations (6 factorial). When the mind grasps more details about an event, it also increases the number of reasonable combinations within that event, enhancing the probability of better understanding its features and particularity. More possible combinations of an event increase its specialty, making the event stand out from similar others. Therefore, an extended ability to manage more items within an array might dramatically increase short-term or active memory. The latter is needed to store verbal data and manage visual data, allocating attention between them. Therefore, the ability of the mind to grasp and manage more combinations within the array improves the ability of short-term memory to store and manage the features of a dataset. These qualities also promote numeracy. Therefore, increasing the size of the focal area of consciousness with four or more items dramatically increases the mental operation with these items concerning their comparison, association, and categorization. Studies have also shown the crucial role of representing and mentally manipulating numbers in improving numerical knowledge (e.g., [43]). The size of the focal area of consciousness for more than three items is crucial for developing math skills. Studies have shown an association of the focal point of attention for up to four items with the development of numeracy in children, which starts developing around the age of 3.5 years.

In this study, we investigated the treatment of children with MLD. Several studies have shown that these disabilities might develop in children with normal IQ in the absence of difficulties in other domains, skills, or abilities (e.g., [1, 5, 9, 10]). This suggests that MLD might develop in children with a typical developmental trajectory when they only lack the insight (or skills) to manage more than three items in the focal point of consciousness. Although these children possess essential cognitive normality, they cannot operate thought items (essential for processing the comparison and association of different items) due to a small focal point.

Landerl et al. [44] found that dyscalculic children have deficits in subitizing. The apparent failure to subitize small numerosities (counting them instead) has been implicated in several cognitive disorders and is associated with dyscalculia (e.g., [45]). Wilson et al. [46] showed that the ability of children improved in a dot enumeration task in subitizing when the researchers conducted a training course (10 weeks) on numeracy with nine children of 7-9 years with DD. According to Gray and Reeve [38], a weak subitizing profile (three dots) might be a diagnostic marker of emerging math difficulties (e.g., dyscalculia) in preschoolers [38].

Cognition occurs, to a large extent, due to shared intentionality in mother-child dyads (e.g., [47]). A study proposed a hypothesis based on neurobiology regarding shared intentionality that occurs in non-perceptual interactions [48]. A study on inter-brain neural synchronization indicated an increase in coordinated neuronal activities in the subjects during cooperative actions when communication via sensory cues was absent [49]. Another hyper-scanning study revealed a greater interpersonal neuronal coordination in subjects involved in solving a problem together than that in individuals working separately on identical tasks [50]. According to Fishburn et al. [50], interpersonal neural synchronization underlies a biological mechanism for shared intentionality. Studies on shared intentionality conducted with adults showed that coordinated actions between unprimed subjects and primed confederates encouraged resolving unintelligible problems (unintelligible for unprimed subjects) without communication via sensory cues provided by primed subjects (confederates) who knew the correct answer [51-53]. The case studies on numeracy in toddlers supported this hypothesis showing non-perceptual interaction in dyads [54, 55].

The electromagnetic field is a factor affecting human-computer interactions. Studies on human-computer interactions have investigated the effects of radiofrequency electromagnetic field exposure on the central nervous system (e.g., [56-58]). However, despite a dramatic increase in the use of mobile electromagnetic devices in daily life, information on the effects of the visible spectrum of the electromagnetic field on cognition is limited. According to Danilov and Mihailova [48, 53], a single harmonic oscillator can induce the neurons (or Coherence Agent from these neurons) of modality-specific gateways in nervous systems of different individuals associated by psychophysiological coherence. Since these neurons react similarly, the neurons of mature organisms show the neurons of the neonates the appropriate response to high-frequency stimulation for inducing long-term potentiation [48]. Therefore, shared intentionality occurs in these organisms. Some studies applied and described this mode of human-computer interaction (e.g., [51-55]). Therefore, human-computer interactions might also stimulate shared intentionality for a child's apperception of more dots in subitizing.

We hypothesized that if shared intentionality promotes cognition from the very beginning of fetal life, this interaction modality can also provide insights into subitizing in older children. We speculated that children might apperceive more items by completing tasks with their mother compared to the number of items they could grasp in subitizing when working independently. Following the above data, we supposed that advanced representation and mental manipulation capabilities of an array increase numerical competence. Therefore, increasing the size of the focal area of attention to four objects facilitates the development of numeracy in children. Although the child is unaware (unconscious) of this training of the focal point, we speculated that, through repetitions, specific neural networks might develop for apprehending a larger number of items. Studies have shown that increasing implicit memory enhances the performance of students in certain tasks [59] and gives them confidence in their decisions [60]. We speculated that even the unaware successful performance of the subjects in subitizing, when cooperating (shared intentionality) with knowers, can train neuronal networks to process the features of the focal area of attention (or elements within the focus of attention). This training might help to develop specific neuronal networks responsible for processing numeracy. Thus, shared intentionality might provide insights into arithmetic principles of associativity and commutativity. It can also ensure implicit memorizing relationships between numbers. In this study, we investigated whether shared intentionality with the mother (stimulated by human-computer interactions) can improve a child's apperception of dots in subitizing, thus, improving her counting skills.

2. Materials and Methods

The training program "Agata-2" of the focal point of attention provided 18 online lessons with several exercises with the different pictures of circular dot stimuli, 10 mm in diameter. The mobile version of the "Agata-2" software showed the subject and her mother the tasks of subitizing. The dyad sat with one smartphone that was placed 0.4 m from the eyes. The design of the experiment stimulated emotional arousal due to the unusual situation of the experiment and rhythmically changing red/purple colors on the mobile screen. The unintelligible tasks also contributed to emotional excitement in the dyad. This software also induced interactional synchrony between the subject and her mother due to the rhythmically changing colors on the phone screen (80 bpm). According to the model of coherent intelligence [48, 52, 54, 55], emotional contagion and interactional synchrony stimulate shared intentionality in dyads. According to Danilov and Mihailova [61], a distance testing procedure also provides emotional stimulation during testing. Empirical studies have shown an increase in the cortisol levels of participants during a public performance, indicating emotional arousal [61]. However, elevated cortisol levels impair cognitive processes. Empirical evidence supports the law of Yerkes and Dodson [62] that optimal but not maximal arousal predicts the highest performance [61]. Similarly, human-computer interaction stimulated shared intentionality for the child's apperception of more dots than she could apprehend alone. The child solved unintelligible tasks by cooperating with her mother [61]. In caregiver-infant dyads, only the caregiver who shares the daily routine with the child can contribute to the emergence of shared intentionality [61].

Four series of stimuli were used in different regimes of the training course. Three to seven dots were displayed on the phone screen in random order using different color groups and exposure times. The training course proceeded with regimes of different complexity; the differences were based on exposure times and the order and color of the dots. Usually, experiments on subitizing measure apprehension of numerosity in subjects based on exposure durations of 100 - 300 ms. These experiments limited the display time of the dots because a longer observation time also allows the subjects to perform counting, which is another mechanism to comprehend numerosity. Adults can covertly count at approximately 268 ms/object [34]. Our training course applied two different exposure durations of 1 s and 2 s. We argue that our training program required this prolonged exposure of the dots to the subject to ensure a shared intentionality effect and promoting the development of the size of the focal area of attention. Children with reduced numeracy need more time to grasp the number of items in a set than normal children. Since the focal area in children with math difficulties is small, they need to count items one by one in a set. We suppose their counting speed is approximately 1 s/object, compared to an adult's 268 ms/object [34]. Therefore, even a long exposure time of 2 s might not be enough for them to correctly count the number of items when more than three items are present. On the other hand, this prolonged exposure helps children (exhibiting shared intentionality with adults) give correct answers in numerosity tasks. Thus, we started the training phase with an exposure of 2 s and then lowered the exposure to 1 s as the perception of numerosity improved.

The last regime differed by modes of answers, i.e., pictures and numbers. The main purpose of this study was to determine an intervention for dyscalculia, and thus, learning would get over once the girl showed an increase in the focal point of attention and attained arithmetic skills similar to her peers. The "case study" was interrupted when the child's math skills reached the same level as that required by the school curriculum. The subject's passage from one training regime to another occurred in two cases: (i) The subject did not show an increase in the focal zone and/or math competence during three training sessions with a repetition of the grasping of the same stimuli. The equal results of the three lessons in a row would suggest that the training had failed and a change in the stimulus was required. (ii) The subject showed an advanced perception of the dots due to training in this regime or an increase in math competence. Then, the subject passed to the next stage, where the task became more difficult.

The instructions for the subject were: "You will be shown successively, for a very short time, pictures on the phone screen, which will contain a varying number of dots on a red background.

Immediately after the exposure, you will report the number of dots that you have apprehended by choosing an answer from four options. The reply ‘I do not know’ is acceptable.” While the subject was performing the tasks, the mother mentally completed the same tasks simultaneously. However, the mother was asked to perform the test mentally only, without communicating with the subject.

After each “Agata-2” training program exercise, the subject was tested on arithmetic competence based on basic math problems. The subject was instructed to report the solution of a basic mathematical problem by choosing a correct answer from four options. After testing math competence, the subject was not provided feedback, and she did not know whether her answer was correct. Thus, although the subject solved basic math problems after each exercise on the size of the focal point of attention, she was not informed about her performance after the math competence testing. She was only asked to solve basic math problems without correcting the results. Therefore, the girl did not learn from this method of testing math competence.

2.1 Intervention Regimes

The course of treatment consisted of the four regimes of human-computer interaction based on the rapid exposure to several pictures with a few dots (called “Subitizing”). This human-computer interaction also stimulated “shared intentionality” in the mother-child dyad for developing the child’s rapid apprehension of these small numbers.

2.1.1 Regime I

The software was used on a smartphone to perform a test for 3 min with the dyad, where black dots were shown on a red background for 2 s. Three to seven dots were arranged on the phone screen in a centrosymmetric order. The subject had to select an answer out of four options in the first and second blocks within 20 s.

2.1.2 Regime II

The software was used to perform a test for 3 min with the subject, where black dots were presented on a red background for 1 s. Three to seven dots were arranged on the phone screen haphazardly, which was different from the arrangement pattern followed in Regime I. The subject had to select an answer out of four options in the first and second blocks within 20 s.

2.1.3 Regime III

Three to seven (black and green) dots were presented in random order on a red background to the subject for 1 s. The dots on each picture were distinguished into two groups (up to three dots in each color group) based on two colors, black and green. The subject had to choose an answer from options in the first and second blocks within 20 s of each trial.

2.1.4 Regime IV

In Regime IV, 10 tasks with black dot sets were performed. The options in the first block of 5 tasks for selecting an answer were four pictures of different sets of dots, not the figures, as in the previous regimes. The exposure time was 1 s. The time limit for answering was 20 s. In this regime, three to seven dots were arranged on the phone screen in a centrosymmetric order as in Regime I.

2.2 Subject

The subject was an eight-year-old girl with cerebral palsy. In the perinatal period, she suffered hypoxic-ischemic brain damage in the form of gliosis atrophic changes in the white matter of the cerebral hemispheres, atrophic changes in the corpus callosum, and ventriculomegaly. In 2014, neuro-sonography (the use of ultrasound to obtain brain images) was performed, and post-hypoxic changes were diagnosed. It revealed periventricular leukomalacia (PVL), which is a softening of white brain tissue near the ventricles. In 2020, kindergarten professionals conducted Psychological Assessment and Speech-Communication Assessments when she was six years old. They found that she showed typical cognitive development. However, the girl later showed difficulties in basic retrieval of math-related facts, i.e., learning simple addition sums. The child did not even understand the meaning of basic arithmetic operations. For example, she gave a sum less than a minor term of the task while solving a basic addition problem: she reported the number 2 to solve the basic fact of 3 addition to 4. She had difficulty stating which of the two numbers was larger. She needed to count even three items in a set one by one. One year of attempts in drilling basic arithmetic facts before this case study was unsuccessful. Before starting the training program, when the girl was 7 years and 10 months old, we used the “Agata-2” software to assess the size of the focal area of attention, which was found to be 2 points.

3. Results

3.1 Regime I

During the four lessons in this regime, the subject’s apperception of dots improved up to three dots. She improved her math competence from three correct (out of 10) solutions to basic math problems with addendum +2 while completing the test. She answered eight answers correctly (out of 10) at the end of this block.

3.2 Regime II

During the three lessons with newly designed images with the dots placed randomly against the background, the subject’s apperception of subitizing did not improve. After completing the test on basic math problems with addendum +2 in the previous regime, the subject was tested with basic math problems with addendum +3. Six problems were correctly solved (out of 10). The performance increased to seven correct answers out of 10 at the end.

3.3 Regime III

The new design on subitizing with pictures showing three to seven colored dots was introduced. The subject’s apperception started with three dots in subitizing. The math competence was six correct solutions out of 10 basic math problems with addendum +3 when completing the test at the beginning of this stage. During the eight lessons, the apperception of dots was improved up to seven dots. At the end of this stage, she showed improved competence in solving math problems with addendum +4 with eight correct answers out of 10, and in math problems with addendum +5, she answered four questions correctly out of 10.

3.4 Regime IV

The task of subitizing was changed. The pictures had all black dots on a red background (as in Regime I). The modes of answer options were changed: four picture options in the first block of five trials and four digits options in the second block. The subject could grasp seven dots in a set (the dots were shown for 1 s). The math competence was four correct solutions out of 10 basic math problems with addendum +5; the test was completed at the beginning of this stage. At the end of this stage, she showed improved competence in solving math problems with addendum +5 and answered eight questions correctly out of 10.

The results of the tests are presented in Table 1 and in Figure 1. The data are presented based on the increasing order of the size of the focus of attention in the 18 lessons on subitizing in human-computer interaction that stimulated shared intentionality. Math competency test scores are presented as a percentage of correct answers out of the 10 tasks (see Table 1 and Figure 1).

Table 1 The results on subitizing.

Regimes of human-computer interaction	**Dots exposure time, seconds**	**The size of the focal point**	**Checking on math problems: Math competency test scores started => finished**			
			X + 2	**X + 3**	**X + 4**	**X + 5**
Regime I						
The 1st lesson.	2	2	30%			
The 2nd-4th lessons.	2	3	90% => 80%			
Regime II						
The 5th-7th lessons.	1	3		60% => 70%		
Regime III						
The 8th-10th lessons.	1	3		60%=>90%		
The 11th lesson.	1	4		80%		
The 12th lesson.	1	4			60%	
The 13th lesson.	1	5			100%	
The 14th lesson.	1	3			80%	
The 15th lesson.	1	7				40%
Regime IV						
The 16th-18th lessons.	1	7				40%=>80%

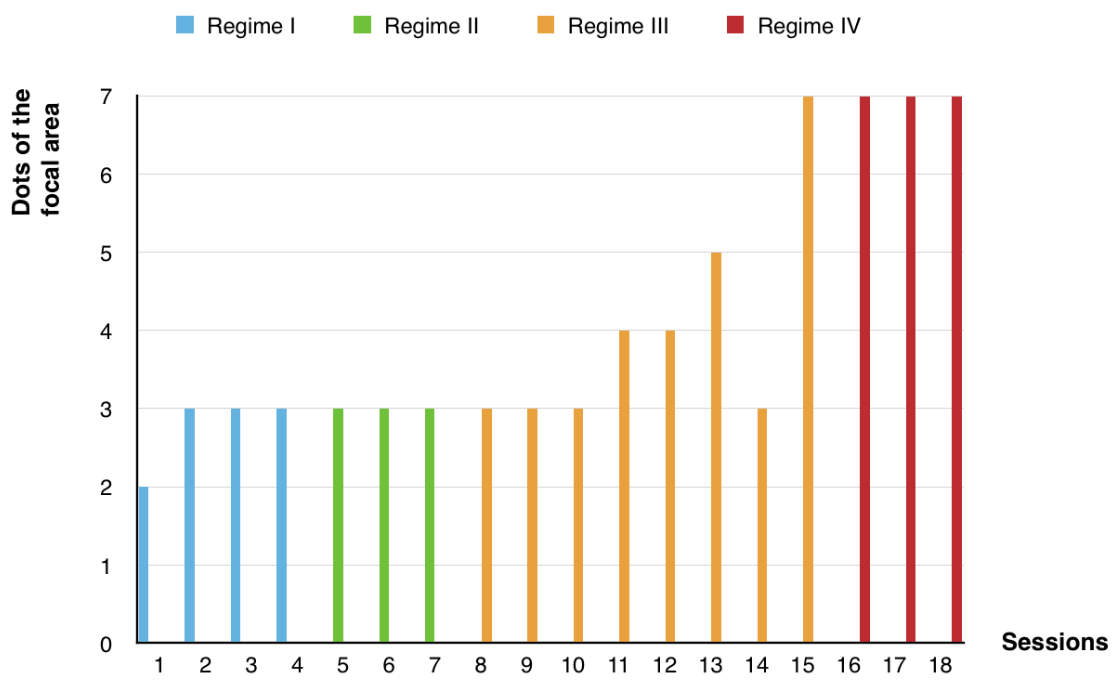

Figure 1 Increasing the size of the focal area of attention in the four regimes using different stimuli in 18 sessions.

4. Discussion

Studies have shown an association of visual-spatial processing with mathematical competence in the early stages of development. Lindsay et al. [63] found worse performance in attention tests and visual-spatial processing in children with dyscalculia compared to the control variables. Based on experimental data, Szucs et al. [64] argued that visual-spatial processing plays an essential role in the development of children in arithmetic. Georges et al. [65] highlighted the importance of spatial-numerical interactions for arithmetic skills.

As mentioned previously, a math-responsive network is established in the bilateral intraparietal sulcus and the inferior temporal cortex [16]. Studies in the field of neuroscience have shown that the bilateral IPS plays an essential role in executive functions [66]. This region might contribute to the accumulation of evidence during decision-making [67]. According to Amalric [16], this network overlaps with the neural correlates of deductive reasoning. The IPS profile provides strong evidence for its role in endogenous attention shifts and features attention shifts rather than exogenous shifts [68]. According to Molenberghs et al. [69], shifts in the attentional focus also activated the IPS but mainly if they were guided endogenously by internal rules of relevance rather than stimulus displacement. Molenberghs et al. [69] found that only the IPS region was activated when the stimulus configuration changed, but the attentional focus remained spatially fixed, i.e., the activity in IPS was mainly related to an endogenous control, even when the spatial focus of attention remained fixed [69]. These findings might indicate an essential role of the IPS region in processing the internal features of the focal area of attention (or elements within the focus of attention). These results also support an association of numerical skills with the size of the focal area of attention. A lower activity of the IPS can reduce the capacity to perform executive functions, accumulate evidence for decision-making, and process features within the attentional focus. Thus, the reduction in activity causes impairment in numerical skills.

Some studies have shown that the neural network responsible for numeracy overlaps with visual and spatial processing regions (e.g., [16]). The theoretical analysis of the data in this study also showed that the expanded size of the focal area of attention with four or more items can dramatically increase the mental operation with these items. These arguments suggest that research on the size of the focal area of attention is required for understanding the development of numerical competence.

The outcome of this case study was interesting. Although the child showed significantly higher scores on arithmetic competence in the written assessment, her oral responses to the same problems did not show the same consistency in the accuracy of the outcome. A robust effect of testing can occur, where participants who learned the materials through testing outperform those who restudied the material [70]. Therefore, a written exam can show a better outcome than an oral exam.

The results of this case study provided a framework for the treatment of dyscalculia. The training was performed while the child was unaware of the process. The procedure to check the answers did not increase her knowledge since she did not receive feedback after testing. Further studies can verify whether combining two methods,- i.e., unconscious training via shared intentionality and classical drilling of basic math problems,- would be more effective for improving arithmetical competence. We propose two training levels for the training course on numerical competence, the first level of “drill-subitizing” and the second level of “math-memorizing.” At the first level of “drill-subitizing,” future studies can design two training exercises for developing the focal point of consciousness in 10 lessons. In Exercise I of this study, mobile software was used to show pictures with colored (black and green) dots on a red background for 2 s in the first block and 3 s in the second block. Three to seven dots were arranged haphazardly on the phone screen. The dots on each picture were distinguished into two groups (up to three dots in each color group) by two colors, black and green. When the subject showed a perception of up to five objects due to training, the subject passed to the next stage, where the task became difficult. In Exercise II, the software was used to present pictures with black dots on a red background for 1 s in the first block (four options of answers were pictures) and 1 s in the second block (four options of answers were numbers). Three to seven dots were arranged on the phone screen in a centrosymmetric order.

“Math-memorizing” can be the second training level of numerical competence in children. There are several memorizing strategies; one of them is drilling basic arithmetic problems. Drill techniques effectively help retain information in long-term memory [71]. Drill activities also help children in understanding mathematical patterns, thus, making it easier for them to make a mental picture while solving mathematical problems [72].

5. Conclusions

In this case study, we investigated an intervention technique to help children overcome a mathematical learning disability. For the first time, the hypothesis of developing numeracy in children by training the apperception of dots in enumeration tasks (subitizing) was tested. Shared intentionality promotes cognition from the onset. Therefore, the study investigated this interaction modality to facilitate insights into expanding apperception for improving numerical competence in an eight-year-old girl. The child was stimulated to apperceive more objects while performing tasks with her mother. The course of treatment consisted of the four regimes of human-computer interaction based on rapid exposure of the subject to multiple pictures with several small dots. The human-computer interaction also stimulated shared intentionality in the mother-child dyad for developing the child's rapid apprehension of these small dots. The results showed that the size of the focal point of attention (consciousness) increased, and numerical competence was developed, which showed an association between the expansion of apperception and the development of numeracy. In this study, we proposed a framework for the treatment of dyscalculia.

Author Contributions

Igor Val Danilov conceived and designed the experiments. Sandra Mihailova reviewed the research design and gave valuable remarks. Igor Val Danilov formulated the hypothesis and wrote the first draft of the manuscript. Igor Val Danilov and Sandra Mihailova improved the text over several iterations.

Competing Interests

The authors have declared that no competing interests exist.

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